Emergent Isotropy-Breaking in Quantum Cosmology

Andrea Dapor[1,*] and Jerzy Lewandowski[1,†]

[1] *Instytut Fizyki Teoretycznej, Uniwersytet Warszawski, ul. Hoża 69, 00-681 Warszawa, Poland*

(Dated: November 2, 2012)

We consider a massive quantum test Klein-Gordon field probing an isotropic quantum cosmological space-time in the background. The result obtained is surprising. It turns out, that despite the isotropy of the quantum gravitational field, the semi-classical metric experienced by a mode of the K-G field is *non-isotropic*. The anisotropy depends on the direction of the momentum of the mode. Specifically, what we do is to derive a semi-classical space-time which emerges to a mode of the field. The method amounts to a comparison between QFT on a quantum background and QFT on a classical curved space-time, giving rise to an emergent metric tensor. The components of the semi-classical metric tensor are calculated from the equation of propagation of the quantum K-G field in the test field approximation. The anisotropies are of a quantum nature: they are proportional to $\hbar$ and "dress" the isotropic classical space-time obtained in the classical limit.

arXiv:1211.0161v1 [gr-qc] 1 Nov 2012

Quantum geometry (QG), is the idea that space-time of general relativity is just a low-energy (i.e., large-scale) effective description of gravity. At a more fundamental level, geometry is believed to be "quantum", and it is only because of the extreme energies that are needed to probe this level of reality that we do not observe any quantum geometry effect in the world around us. In fact, phenomenological arguments suggest that the energies at which the quantum nature of geometry cannot be disregarded are in the Planck regime ($E_{\rm Pl} \approx 1.22 \times 10^{28}\ eV$, corresponding to Planck length $\ell_{\rm Pl} \approx 1.62 \times 10^{-35}\ m$). As to the explicit description of quantum geometry, some proposals exist, such as Geometrodynamics [1–3] and Connessiodynamics [4–6] (also known as Loop Quantum Gravity (LQG)). At the theoretical level, both these theories present difficulties due to the non-perturbative structure required for background-independence; nevertheless, in recent years LQG has seen a great development, and is on the verge of providing a possible complete description of the dynamics of quantum geometry. Accordingly, it is now possible to start using the theory to compute transition amplitudes, and possibly to predict observable effects which in principle can be used to falsify it.

A most successfull application of the theory is in the cosmological sector [7, 8], in which case the common name is Loop Quantum Cosmology (LQC).[1] Studying the dynamics of this theory, one finds that the classical singularity is removed [9–11], being in fact replaced by a bounce, thus solving one of the greatest problems of classical general relativity (at least in the cosmological sector).

Recently, there has been some interest in a new tool available in the context of quantum cosmological models, LQC in particular, namely that of quantum test fields probing quantum space-time. This concept was introduced in [12] (later developed in [13], and applied in [14] to the study of primordial quantum perturbations and possible observable effects in the CMB), where the authors considered the quantum Klein-Gordon test field propagating on a quantum space-time of the cosmological type in the background. The goal of the works was to derive quantum matter test fields and to "probe" with them the quantum nature of the underlying geometry. As a result, quantum equations of quantum test fields propagating in quantum space-times were derived. Taking the classical limit in the gravitational degrees of freedom (but leaving the quantum matter field still quantum) gives the standard equations of QFT in a certain classical space-time, say $g_{\rm class}$, in the background. Moreover, it was formulated another, quite natural *semi*-classical limit, sensitive to the quantum corrections coming from the quantum nature of the gravitational field [12]. In this limit the equations satisfied by the quantum matter test field on the quantum background again take the form of the equations of QFT, however in a different, semi-classical space-time. This semi-classical metric differs from $g_{\rm class}$ by corrections depending on quantum fluctuations of the quantum geometry operators, therefore it was called quite accurately in [14] a "dressed metric"

$$g_{\rm dress} \ = \ g_{\rm class} + O(\hbar). \qquad (1)$$

The dressed metric is a new classical metric[2] "felt" by the matter field. Thus, it became clear that effects of the interaction between quantum matter (in the test field approximation) and quantum geometry could be understood by comparing the dressed metric with the expected classical metric. In [12], the dressed metric was computed

*Electronic address: adapor@fuw.edu.pl

†Electronic address: jerzy.lewandowski@fuw.edu.pl

[1] It is to be said that LQC does not directly descend from LQG, but it is rather a finite degree of freedom model of the LQG-like quantization in the phase space of cosmology, obtained as a sector of the phase space of general relativity by restricting to homogeneous space-times.

[2] It should be clear that $g_{\rm dress}$ is classical, in the sense that it is a metric tensor whose coefficients are functions, not operators. However, this metric contains quantum corrections and in general does not satisfy classical Einstein equations.

for a test massless K-G field in a isotropic quantum space-time of the FRW type. It was observed that the metric was independent of the momentum $\vec{k}$ of considered mode. Therefore, in [13] the procedure was applied to the same quantum field on a Bianchi I space-time, with the hope that in this case the dressed metric could depend on the mode's energy and direction of propagation: if that were the case, quanta of different energies would move at different speeds (because they would "feel" different dressed metrics), and thus a violation of Lorentz symmetry would be observed. The dressed metric for modes of the massless K-G field was computed: it turned out that such a violation is not present, since the dressed metric is the same for all modes of the matter field (and this result also applies to the subcase case of FRW quantum space-time). However, a major limitation of that work was present, namely, we were not able to identify the dressed metric in the case of a *massive* K-G field. This is the problem we solve in the current work, in the case of a quantum test massive K-G field on an isotropic quantum space-time.

Below, we review the concept of dressed metric. Next we calculate the dressed metric for the quantum test massive K-G field on an isotropic quantum FRW space-time. The dressed metric we find is not any longer isotropic. In fact it is of the Bianchi I type. Therefore, the major result is that, in this massive case, a Lorentz-violation indeed takes place, because the resulting dressed metric depends on the direction of the specific particle: we can thus say that, if the quantum nature of the space-time is taken into account, then the presence of quantum massive matter produces an *isometry breaking*. Our derivation is quite general and it is independent of a specific choice of the quantum model of space-time: it may well be the original WdW approach, it may be LQC. For the sake of clarity, we refer to the LQC models. The same is true for the matter content of the background quantum space-time and the "choice of time": our result is mostly independent of those, but to be explicit we will use the "irrotational dust" introduced recently in [15] (see [16] for further reference).

I. CLASSICAL THEORY AND QUANTIZATION

We consider the theory described by the action

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{8\pi G}R - \frac{1}{2}M\left(g^{\mu\nu}\partial_\mu T\partial_\nu T + 1\right)\right] + S_M \tag{2}$$

The first term is the usual Hilbert action for geometry, the second term describes *irrotational dust*, and S_M stands for other forms of matter, which eventually will be the Klein-Gordon field. The explicit choice of irrotational dust as part of the matter content is useful to carry out the deparametrization of the theory with respect to T. In other words, following [15], we will choose T to represent the physical time.

Since we are interested in the FRW sector of the theory, we consider the symmetry-reduced class of metrics

$$g_{\mu\nu} = -dt^2 + a(t)^2\left(dx^2 + dy^2 + dz^2\right) \tag{3}$$

and the dust field is homogeneous on the spatial slices (that is, $\partial_i T = 0$ for $i = 1, 2, 3$). The canonical analysis of the system produces a kinematical phase space $\Gamma_{\rm kin}$ coordinatized by the degrees of freedom of the geometry and the matter. Specifically, the momentum conjugate to the dust configuration variable T is given by

$$p_T = a^3 M\dot{T} \tag{4}$$

The dynamics is completely constrained by the only constraint that survives the symmetry-reduction: the homogeneous part of the total Hamiltonian constraint

$$C = \int d^3x H \tag{5}$$

where

$$H = H_G + H_M + p_T \tag{6}$$

Here, H_G and H_M are respectively the geometry part and the matter part of the Hamiltonian. Proceeding with Dirac quantization of constrained theories, we define the kinematical Hilbert space as $\mathcal{H}_{\rm kin} = \mathcal{H}_G \otimes \mathcal{H}_M \otimes L^2(\mathbb{R}, dT)$, where for now the geometric and matter Hilbert spaces remain unspecified. Formally, the quantum operator on $\mathcal{H}_{\rm kin}$ corresponding to H is then

$$\widehat{H} = \widehat{H}_G + \widehat{H}_M - i\hbar\partial_T \tag{7}$$

so physical states $\Psi \in Ker\widehat{H}$ are those $\Psi \in \mathcal{H}_{\rm kin}$ such that

$$i\hbar\partial_T\Psi(v, q_M, T) = \left[\widehat{H}_G + \widehat{H}_M\right]\Psi(v, q_M, T) \tag{8}$$

where v is the variable that parametrizes the spectrum of some distingushed operator in $\mathcal{H}_G$ (see next section) and q_M denotes the collecion of matter variables which coordinatize joint spectrum of suitable set of distingushed operators in $\mathcal{H}_M$.

The form of $\widehat{H}_G$ (and its well-definiteness) depends on the specific quantum theory of gravity, whereas $\widehat{H}_M$ involves both geometrical and matter operators (unless the matter is quantized in a background-independent way). For the sake of clarity, we will consider the LQC quantization (also known as "polymer quantization") of the geometrical part, presented in the next section. However, it is important to observe that the results we will find do not rely on this specific choice, and can be repeated for any other proposal.

II. POLYMER QUANTUM FRW SPACE-TIME

Let us focus on the geometric sector Γ_G of the phase space. In the symmetry-reduced model of FRW type, Γ_G

is coordinatized by the scale factor a and its conjugate momentum p_a, satisfying the Poisson relation $\{a, p_a\} = 1$. a is positive, so to extend the topology of Γ_G to $\mathbb{R}^2$ we perform a canonical transformation to a new set of variables: the *oriented volume* $v := a^3/\alpha$ (where $\alpha = 2\pi\gamma\sqrt{\Delta}\ell_{\rm Pl}^2$, with γ is Barbero-Immirzi parameter and Δ is the so-called "area gap" given by $\Delta = 4\sqrt{3}\pi\gamma\ell_{\rm Pl}^2$) and its conjugate momentum b, satisfying $\{v, b\} = 2$. With respect to these variables, the gravitational Hamiltonian is

$$H_G = \frac{3\pi G}{2\alpha} b^2 |v| \tag{9}$$

At the quantum level, polymer representation of the Poisson albebra of v and b is characterized by the Hilbert space $\mathcal{H}_G = L^2(\bar{\mathbb{R}}, d\mu_{\rm Bohr})$, where $\bar{\mathbb{R}}$ is the Bohr compactification of the real line and $d\mu_{\rm Bohr}$ the Haar measure on it [17]. On this Hilbert space it is defined the action of operators $\widehat{v}$ and of $\widehat{N} := \widehat{\exp(ib/2)}$ (the exponentiated version of b, since due to discreteness of space the infinitesimal action is not available):

$$\widehat{v}|v\rangle = v|v\rangle, \qquad \widehat{N}|v\rangle = |v+1\rangle \tag{10}$$

where $\{|v\rangle\}$ is the basis of eigenstates of $\widehat{v}$, satisfying orthonormality with respect to Kronecher delta $\langle v|v'\rangle = \delta_{v,v'}$. Using a natural symmetric ordering, the gravitational Hamiltonian is implemented as an operator on $\mathcal{H}_G$:

$$\widehat{H}_G = -\frac{3\pi G}{8\alpha}\sqrt{|\widehat{v}|}\left(\widehat{N}^2 - \widehat{N}^{-2}\right)^2\sqrt{|\widehat{v}|} \tag{11}$$

The properties of this operator are studied in [18], but for our purposes it is sufficient to say that it well-defined and essentially self-adjoint.

III. QFT ON QUANTUM SPACETIME

We now need to choose the matter part of the system. We will consider a scalar K-G field ϕ with action

$$S_M = \frac{1}{2}\int d^4x\sqrt{-g}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2\right) \tag{12}$$

Plugging the ADM metric with lapse $N = 1$ in this, and performing canonical analysis, we obtain the matter Hamiltonian:

$$H_M = \sum_{\vec{k}\in\mathcal{L}} H_{\vec{k}} = \frac{1}{2a^3}\sum_{\vec{k}\in\mathcal{L}}\left[p_{\vec{k}}^2 + \left(a^4|\vec{k}|^2 + a^6 m^2\right)q_{\vec{k}}^2\right] \tag{13}$$

where $\mathcal{L}$ is the three-dimensional lattice spanned by $(k_1, k_2, k_3) \in (2\pi\mathbb{Z})^3$. Notice that H_M is nothing but the Hamiltonian of a collection of decoupled harmonic oscillators with a geometry-dependent (and thus time-dependent) frequency

$$\omega_{\vec{k}}^2(a) = \left(a^4|\vec{k}|^2 + a^6 m^2\right) \tag{14}$$

For our purposes, it is sufficient to consider a single mode $q_{\vec{k}}$, since we are going to disregard any matter back-reaction in the so-called "test-field approximation" (see later). However, it should be said that quantization of the full system would require to take into account all modes. In doing so, renormalization of the UV limit is a crucial element: without it, expressions such as equation (13) are entirely formal. Quantizing such an infinite-dimensional system is not at all straightforward, and leads to the whole topic of QFT in curved space-time. On the other hand, as long as one is interested only in a single mode (or a finite set), quantization is on the line of quantum harmonic oscillator: the Hilbert space of matter is given by $\mathcal{H}_M = L_2(\mathbb{R}, dq_{\vec{k}})$, and the dynamical variables $q_{\vec{k}}$ and $p_{\vec{k}}$ are promoted to operators on it, $\widehat{q}_{\vec{k}} = q_{\vec{k}}$ and $\widehat{p}_{\vec{k}} = -i\hbar\partial/\partial q_{\vec{k}}$.

At this quantum level, the dynamics is described by the Schroedinger equation (8):

$$i\hbar\partial_T\Psi(v, q_{\vec{k}}, T) = \left[\widehat{H}_G + \frac{\hat{a}^{-3}}{2}\left(\widehat{p}_{\vec{k}}^2 + \widehat{\omega}_{\vec{k}}^2\widehat{q}_{\vec{k}}^2\right)\right]\Psi(v, q_{\vec{k}}, T) \tag{15}$$

having defined the geometric operator $\widehat{\omega}_{\vec{k}}(a) := \omega_{\vec{k}}(\widehat{a})$.

Now, we use the *test-field approximation*, i.e. the fact that the matter back-reaction is disregarded. This translates mathematically by saying that the total state $\Psi(v, q_{\vec{k}}, T)$ decomposes as a simple tensor product

$$\Psi(v, q_{\vec{k}}, T) = \Psi_o(v, T) \otimes \psi(q_{\vec{k}}, T) \tag{16}$$

for any time T, where the geometry state Ψ_o obeys the "unperturbed" Schroedinger equation $i\hbar\partial_T\Psi_o = \widehat{H}_G\Psi_o$. In other words, the matter part and the gravity part are disentangled, and evolution of the gravity part does not take into account the presence of matter. Plugging (16) in (15) and projecting on Ψ_o itself, one is left with a Schroedinger equation for matter only:

$$i\hbar\partial_T\psi(q_{\vec{k}}, T) = \frac{1}{2}\left(\langle\hat{a}^{-3}\rangle_o\widehat{p}_{\vec{k}}^2 + \langle\hat{a}^{-3}\widehat{\omega}_{\vec{k}}^2\rangle_o\widehat{q}_{\vec{k}}^2\right)\psi(q_{\vec{k}}, T) \tag{17}$$

where $\langle\widehat{A}(T)\rangle_o := \langle\Psi_o(v,0)|\widehat{A}(T)|\Psi_o(v,0)\rangle$ for every geometrical operator $\widehat{A}$ (time-evolution is moved from the state Ψ_o to the operators, $\widehat{A}(T) = e^{i\widehat{H}_G T/\hbar}\widehat{A}e^{-i\widehat{H}_G T/\hbar}$, thereby realising Heisenberg picture for the gravitational sector (or rather the interaction picture from the point of view of the coupling with the K-G field).

A. The concept of Dressed Metric

The heart of the new approach to matter on *quantum* space-time, is the observation that equation (17) is surprisingly similar to the Schroedinger equation for the states of the quantum field ϕ on a suitably defined *classical* space-time. Let the geometry be classically described

by a metric $\bar{g}_{\mu\nu}$ of the FRW form:

$$\bar{g}_{\mu\nu}dx^\mu dx^\nu = -\bar{N}^2 dt^2 + \bar{a}^2(dx^2+dy^2+dz^2) \tag{18}$$

One can build regulard QFT on such a curved space-time, obtaining for (a single mode $\vec{k}$ of) a scalar field ϕ of mass m the following effective Schroedinger equation:

$$i\hbar\partial_t\psi(q_{\vec{k}},t) = \frac{\bar{N}}{2\bar{a}^3}\left(\widehat{p}^2_{\vec{k}} + \bar{\omega}^2_{\vec{k}}\widehat{q}^2_{\vec{k}}\right)\psi(q_{\vec{k}},t) \tag{19}$$

Comparison with (17) leads the following system of equations:

$$\bar{N}/\bar{a}^3 = \langle\hat{a}^{-3}\rangle_o, \quad \bar{N}\bar{a}|\vec{k}|^2 = \langle\hat{a}\rangle_o|\vec{k}|^2, \quad \bar{N}\bar{a}^3m^2 = \langle\hat{a}^3\rangle_o m^2 \tag{20}$$

This is a system of three equations for two unknowns ($\bar{N}$ and $\bar{a}$), and in general it has no solution for $m \neq 0$. However, in the case $m = 0$, the last equation drops out. Indeed, for a massless K-G field one has a unique solution:

$$\bar{N} = [\langle\hat{a}\rangle_o^3\langle\hat{a}^{-3}\rangle_o]^{1/4}, \qquad \bar{a} = [\langle\hat{a}\rangle_o/\langle\hat{a}^{-3}\rangle_o]^{1/4} \tag{21}$$

We can then rewrite (18) explicitely:

$$\bar{g}_{\mu\nu}dx^\mu dx^\nu = [\langle\hat{a}\rangle_o^3\langle\hat{a}^{-3}\rangle_o]^{1/2}\times \\ \times\left(-dt^2 + \frac{1}{\langle\hat{a}\rangle_o\langle\hat{a}^{-3}\rangle_o}(dx^2+dy^2+dz^2)\right) \tag{22}$$

We managed to express the effective metric in terms of mean values of geometrical operators on the quantum state of geometry Ψ_o. This object was defined first in [12] and next it has been called the *dressed metric* in [14], as it represents the effective classical geometry on which the $\vec{k}$-mode of the matter field lives, in the sense that one can describe the evolution of such a mode on quantum geometry in terms of the same mode propagating on the classical dressed space-time $\bar{g}_{\mu\nu}$. The fact that $\bar{g}_{\mu\nu}$ does not show any dependence on $\vec{k}$ is a hint that no symmetry-breaking takes place: all quanta of matter "feel" the same effective metric, probe the same "eigenstate" of geometry. The proof that this is indeed true was given via dispersion relation analysis in [13] (in the more general case of Bianchi I quantum geometry). Someone could thing that this result is trivial, because the back-reaction of the field on the geometry was disregarded. To argue that the result is not trivial, consider the following two points: (i) The dressed metric (22) is *not* the classical limit metric $\tilde{g}_{\mu\nu}dx^\mu dx^\nu = -dt^2 + \langle\hat{a}\rangle_o^2(dx^2+dy^2+dz^2)$; (ii) The back-reaction features also for the purely classical theory, nonetheless nobody expects that that the classical back-reaction generates a Lorentz violation. So back-reaction is not what we want to study.[3] What we want to study is the effect of the quantum nature of the geometry on the *test* quantum matter field. The reader will see in a moment that, in the case of massive K-G field, this effect is emergent isotropy breaking.

[3] Moreover, it does not seem consistent to take into account only the back-reaction of a single mode: one should indeed consider all modes.

B. Massive case

We saw above that in the case of the massive K-G field, we could not find a dressed metric. However, our anzatz (18) for $\bar{g}_{\mu\nu}$ had a drawback: the assumption of space isotropy. The isotropy seemed natural, because the underlying quantum space-time is isotropic (in the sense that it is obtained by the quantization of isotropic FRW metric). But in fact, the symmetry of the system consisting of the isotropic quantum gravitational field *and* a quantum mode with momentum $\vec{k}$ is the *axial symmetry* preserving $\vec{k}$. Therefore, to find the dressed metric it is resonable to enlarge the number of degrees of freedom that describe $\bar{g}_{\mu\nu}$, and consider the Bianchi type I metrics with the suitable axial symmetry. Consider first the most general Bianchi I metric:

$$\bar{g}_{\mu\nu}dx^\mu dx^\nu = -\bar{N}^2dt^2 + \sum_i \bar{a}_i^2(dx^i)^2 \tag{23}$$

The regular QFT on this curved space-time produces the following Schroedinger equation:

$$i\hbar\partial_t\psi(q_{\vec{k}},t) = \frac{\bar{N}}{2\bar{a}_1\bar{a}_2\bar{a}_3}\left(\widehat{p}^2_{\vec{k}} + \bar{\omega}^2_{\vec{k}}\widehat{q}^2_{\vec{k}}\right)\psi(q_{\vec{k}},t) \tag{24}$$

where

$$\bar{\omega}^2_{\vec{k}} = (k_1\bar{a}_2\bar{a}_3)^2 + (k_2\bar{a}_3\bar{a}_1)^2 + (k_3\bar{a}_1\bar{a}_2)^2 + (\bar{a}_1\bar{a}_2\bar{a}_3)^2m^2 \tag{25}$$

Again, comparison with (17) gives a system of equations:

$$\frac{\bar{N}}{\bar{a}_1\bar{a}_2\bar{a}_3} = \langle\hat{a}^{-3}\rangle_o, \qquad \bar{N}\bar{a}_1\bar{a}_2\bar{a}_3 = \langle\hat{a}^3\rangle_o \tag{26}$$

$$\bar{N}\frac{\bar{a}_2\bar{a}_3}{\bar{a}_1} = \bar{N}\frac{\bar{a}_3\bar{a}_1}{\bar{a}_2} = \bar{N}\frac{\bar{a}_1\bar{a}_2}{\bar{a}_3} = \langle\hat{a}\rangle_o \tag{27}$$

Now, in the generic case of $k_1, k_2, k_3 \neq 0$ there are five equations for four unknowns, so the system is overcomplete and in fact in general has no solution. But we were going to consider the metrics of the axial symmetry around the axis defined by $\vec{k}$. Those metrics are characterised by the diagonal directions such that promoted for the axis of the coordinate system make $\vec{k} = (0, 0, k_3)$. In this case, the only equation of (27) that does not drop is $\bar{N}\bar{a}_1\bar{a}_2/\bar{a}_3 = \langle\hat{a}\rangle_o$, which together with the second of (26) gives $\bar{a}_3 = \sqrt{\langle\hat{a}^3\rangle_o/\langle\hat{a}\rangle_o}$. The remaining two variables must satisfy $\bar{a}_1\bar{a}_2 = \sqrt{\langle\hat{a}^3\rangle_o\langle\hat{a}\rangle_o}/\bar{N}$. There are an infinity of pairs of values that satisfy this relation, but the one that is axially symmetric is of course the one with $\bar{a}_1 = \bar{a}_2$. This fixes a unique solution

$$\bar{N} = \sqrt{\langle\hat{a}^3\rangle_o\langle\hat{a}^{-3}\rangle_o}, \qquad \bar{a}_3 = \sqrt{\langle\hat{a}^3\rangle_o/\langle\hat{a}\rangle_o} \tag{28}$$

$$\bar{a}_1 = \bar{a}_2 = [\langle\hat{a}\rangle_o/\langle\hat{a}^{-3}\rangle_o]^{1/4} \tag{29}$$

The dressed metric is thus of the Bianchi I type (23), despite the fact that the underlying quantum geometry is FRW. The fact that the propagation of massive particles on a quantum FRW space-time can be interpreted equivalently as a propagation of massive particles on a classical Bianchi I space-time shows that, at the semi-classical level, an *isotropy breaking* takes place. At first sight, it may not be so surprising that a preferred direction is selected: after all, we are considering a specific particle, moving in a specific direction (namely, $\vec{k}$), and thus it is natural to expect that the back-reaction should have a direction-dependent effect on the geometry. This is certainly true, but one has to remember that in our analysis we completely disregarded any back-reaction of matter on geometry. In other words, the space-time does not know anything about the existence of the propagating particle. Indeed, what is deformed is the effective metric *felt* by the particle, not the space-time itself: an external observer would still measure a space-time of the FRW type. Having said this, however, if the observer measures characteristics intrinsic to the particle (such as its velocity), then the form of the effective dressed metric shall have an observable effect on them, for example producing a deformation in the dispersion relation.

Interestingly, a certain degree of symmetry remains: the scale factors in the directions orthogonal to the direction of propagation of the particle are unchanged with respect to the massless case (i.e., $\bar{a}_1 = \bar{a}_2 = \bar{a}$). What changes is the scale factor in the direction $\vec{k}$ (here the z-direction) and the lapse function. In particular, if we denote the FRW lapse function in (21) by $\bar{N}_o$, then we find

$$\bar{N} = \alpha \bar{N}_o, \qquad \bar{a}_3 = \alpha \bar{a} \tag{30}$$

where

$$\alpha := \left[\frac{\langle \hat{a}^3 \rangle_o^2 \langle \hat{a}^{-3} \rangle_o}{\langle \hat{a} \rangle_o^3} \right]^{1/4} \tag{31}$$

As expected, α reduces to 1 if we can write $\langle \hat{a}^3 \rangle_o = \langle \hat{a} \rangle_o^3$ (as would happen if the geometry were completely classical), thus recovering the FRW case (more precisely, we would get the classical limit metric $\bar{a}_1 = \bar{a}_2 = \bar{a}_3 = \langle \hat{a} \rangle_o$). This proves that the deformation of the symmetry is an effect of the quantum nature of the geometry, confirming that at 0th order in the back-reaction there is no classical effect on the geometry.

It is possible to expand $\alpha = 1 + \delta\alpha$. To do this, define the quantities $\delta A := (\langle \hat{a}^3 \rangle_o - \langle \hat{a} \rangle_o^3)/\langle \hat{a} \rangle_o^3$ and $\delta B := (\langle \hat{a}^{-3} \rangle_o - \langle \hat{a} \rangle_o^{-3})/\langle \hat{a} \rangle_o^{-3}$, which measure the "non-classicality" of the geometry in a way similar to the usual variance. Then, we can write

$$\begin{aligned} \alpha &= \left[(1 + 2\delta A + \delta A^2)(1 + \delta B) \right]^{1/4} \approx \\ &\approx 1 + \frac{1}{2}\delta A + \frac{1}{4}\delta B \end{aligned} \tag{32}$$

having retained only the first order corrections in δA and δB. Recalling that $\hat{a}$ depends on time T, we can identify two limits: close to the bounce $\langle \hat{a} \rangle_o \to 0$, while at large times $\langle \hat{a} \rangle_o \to \infty$. Thus, in these two regimes the correction α is dominated by either δA or δB, respectively. However, for large times the quantum nature of geometry can be neglected [11], so one can argue that δB (as well as δA) vanishes in the classical gravity regime: in other words, a deviation from the FRW metric is present only at early times, i.e. in the vicinity of the bounce.

IV. CONCLUSIONS

In this work, we reviewed the concept of dressed metric, applied it to quantum cosmology of the FRW type, and we saw how to extend its applicability to massive fields by extending the kinematical family of the effective space-time $\bar{g}_{\mu\nu}$. For definiteness (and for possible further computations), for the quantum background metric we explicitely chose the isotropic LQC space-time coupled to homogeneous irrotational dust (which is used as the deparametrising time). However, it is clear that the general method works also for other well-defined quantum geometries, and for any other choice of matter for physical time that produces a dynamical equation of the form (8). Note that, for the choice of time usual in LQC literature (namely, a homogeneous massless K-G background field T - see [19, 20] for further reference), one obtains a Schroedinger-like equation of the form

$$i\hbar \partial_T \Psi(v, q_M, T) = \sqrt{\widehat{H}_G^2 - 2\widehat{H}_M} \Psi(v, q_M, T) \tag{33}$$

In this case, it is necessary to use an operator expansion, which produces an approximate dynamical equation:

$$i\hbar \partial_T \Psi(v, q_M, T) = \widehat{H}_G - \widehat{H}_G^{-1/2} \widehat{H}_M \widehat{H}_G^{-1/2} \Psi(v, q_M, T) \tag{34}$$

A part from this modification, the argument then proceeds as above.

Independently of the choices we adopted, the qualitative behaviour does not change: for massless fields, the dressed metric is independent on energy and momentum of the particle; but once a massive field is taken into account, an isotropy-breaking takes place, and the dressed metric assumes the form of a Bianchi I metric deformed in the direction of motion of the particle. Interestingly, this deformation is independent on the magnitude of $\vec{k}$ or the mass m (i.e., it does not depend on the energy of the particle), but depends only on the direction of propagation: so in principle this effect is present not only for high energy particles (such as GRB), but for *any* moving massive particle! Nonetheless, the "dressing" of the classical limit metric on the quantum mode requires that the gravity is strongly non-classical, and hence the deformation is expected to take place only in the vicinity of the bounce, i.e. in the primordial past. This explains why nowadays we do not observe any such effect around us. Nevertheless, it is conceivable that it could have

played an important role in the conditions at early times (in the sense of initial quantum perturbations), possibly leaving a signature in the cosmic microwave background (CMB) or influencing the formation of structures.

V. ACKNOWLEDGMENTS

This work was partially supported by the grant 182/N-QGG/2008/0 (PMN) of Polish Ministerstwo Nauki i Szkolnictwa Wyższego.

[1] B. S. DeWitt, *Quantum theory of gravity. I. The canonical theory*, Phys. Rev. **160**, 111348 (1967).

[2] B. S. DeWitt, *Quantum theory of gravity. II. The manifestly covariant theory*, Phys. Rev. **162**, 1195238 (1967).

[3] J. A. Wheeler, *Geometrodynamics* (Academic Press, New York, 1962).

[4] A. Ashtekar, J. Lewandowski, *Background independent quantum gravity: A status report*, Class. Quant. Grav. **21**, R53-R152 (2004).

[5] T. Thiemann, *Introduction to Modern Canonical Quantum General Relativity* (Cambridge University Press, Cambridge, 2007).

[6] C. Rovelli, *Quantum Gravity* (Cambridge University Press, Cambridge 2004).

[7] A. Ashtekar, P. Singh, *Loop quantum cosmology: A status report*, Class. Quantum Grav. **28**, 213001 (2011).

[8] M. Bojowald, Living Rev. Relativity **8**, 11 (2005).

[9] A. Ashtekar, T. Pawlowski, P. Singh, *Quantum nature of the big bang*, Phys. Rev. Lett. **96**, 141301 (2006).

[10] A. Ashtekar, T. Pawlowski, P. Singh, *Quantum nature of the big bang: An analytical and numerical investigation I*, Phys. Rev. D **73**, 124038 (2006).

[11] A. Ashtekar, T. Pawlowski, P. Singh, *Quantum nature of the big bang: Improved dynamics*, Phys. Rev. D **74**, 084003 (2006).

[12] A. Ashtekar, W. Kaminski, J. Lewandowski, *Quantum field theory on a cosmological, quantum space-time*, Phys. Rev. D **79**, 0644030 (2009).

[13] A. Dapor, J. Lewandowski, Y. Tavakoli, *Lorentz symmetry in QFT on quantum Bianchi I space-time*, Phys. Rev. D **86**, 064013 (2012).

[14] I. Agullo, A. Ashtekar, W. Nelson, *A Quantum Gravity Extension of the Inflationary Scenario*, [arXiv:1209.1609].

[15] V. Husain, T. Pawlowski, *Time and a Physical Hamiltonian for Quantum Gravity*, Phys. Rev. Lett. **108**, 141301 (2012).

[16] K. Giesel, T. Thiemann, *Algebraic quantum gravity (AQG): IV. Reduced phase space quantization of loop quantum gravity*, Class. Quantum Grav. **27**, 175009 (2010).

[17] A. Ashtekar, M. Bojowald, J. Lewandowski, *Mathematical structure of loop quantum cosmology*, Adv. Theor. Math. Phys. , 233-268 (2003).

[18] W. Kaminski, J. Lewandowski, *The flat FRW model in LQC: self-adjointness*, Class. Quant. Grav. 25, 035001 (2008).

[19] M. Domagala, K. Giesel, W. Kaminski, J. Lewandowski, *Gravity quantized: Loop quantum gravity with a scalar field*, Phys. Rev. D. 82, 104038 (2010).

[20] M. Domagala, M. Dziendzikowski, J. Lewandowski, *The polymer quantization in LQG: massless scalar field*, [arXiv:1210.0849]